Coordinates, Eigenvectors, and Diagonalization

Juniper Bahr

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Contents

1 Coordinates Basics

1.1 What is a basis?

Let V be a vector space over the field F.

Definition 1.1. The vectors $\{v_1, \ldots, v_n\}$ are said to be *linearly independent* if for *any* $c_1, \ldots, c_n \in F$,

$$\sum_{i=1}^{n} c_i v_i = 0 \implies c_1 = c_2 = \cdots = 0$$

This means the *only* way of "reaching" 0 using a linear combination of $v_1, \ldots, v_n$ is by taking exactly zero of every one of those vectors. It rules out things like $v_1 - 2v_2 + 4v_3 + 0v_4 = 0$.

Very informally, linear independence is a way of saying we don't have "too many vectors" in a set.

What about the set of vectors we *can* "reach" from $v_1, \ldots, v_n$?

Definition 1.2. The *span* of the set of vectors $\{v_1, \ldots, v_n\}$ is the set of all linear combinations of $v_1, \ldots, v_n$, given as follows:

$$\text{span}(\{v_1, \ldots, v_n\}) = \left\{ \sum_{i=1}^{n} c_i v_i \mid c_1, \ldots, c_n \in F \right\}$$

The span of a set of vectors is the set of all linear combinations of those vectors. So the span of v_1, v_2 is a set that includes things like $v_1 + v_2$, $v_1 - 2v_2$, $0v_1 - 7v_2$, and so on.

Let's put these concepts together!

Definition 1.3. If $\{v_1, \ldots, v_n\}$ are linearly independent, and $\text{span}(\{v_1, \ldots, v_n\}) = W$ for some subspace $W \subseteq V$, then we say $\beta = \{v_1, \ldots, v_n\}$ is a *basis* for W.

The number of vectors in a basis is *independent of what basis you pick!* This is a little surprising perhaps. It means *any* basis of $\mathbb{R}^2$ has exactly two vectors in it, since we know $\{e_1, e_2\}$ is a basis for $\mathbb{R}^2$ (and it has two vectors in it).

1.2 What are coordinates?

Suppose $x \in V$ and $\beta = \{v_1, \ldots, v_n\}$ is a basis for V.

Since $\text{span}(\beta) = V \ni x$, we know that $x = \sum_{i=1}^{n} c_i v_i$ for some $c_1, \ldots, c_n \in F$. But maybe there are lots of scalars that work instead of just $c_1, \ldots, c_n$, right? Actually no! So let's make a definition:

Definition 1.4. Let $x \in V$ and $\beta = \{v_1, \dots, v_n\}$ be a basis for V. Then the *coordinate vector of x with respect to the basis β*, denoted $[x]_\beta$ is given by

$$[x]_\beta = (c_1, \dots, c_n)$$

where $c_1, \dots, c_n$ are the **unique** scalars in F satisfying

$$x = c_1 v_1 + \cdots + c_n v_n$$

Ok so why is there only one set of scalars that works there?

Proposition 1.5. *Suppose $x \in V$ with $\beta = \{v_1, \dots, v_n\}$ a basis for V and*

$$x = \sum c_i v_i = \sum d_i v_i$$

for scalars $c_1, \dots, c_n, d_1, \dots, d_n \in F$.

Then $c_i = d_i$ for each $1 \leq i \leq n$! That is, the set of scalars are unique.

Proof. If $x = \sum c_i v_i = \sum d_i v_i$, then we can write zero as follows:

$$0 = x - x = \sum (c_i - d_i) v_i$$

but since $\{v_1, \dots, v_n\}$ is a linearly independent set of vectors, $c_i - d_i = 0$ for each i, and thus $c_1 = d_1, c_2 = d_2, \dots, c_n = d_n$ as we wanted to show! □

So when I defined coordinates, it was reasonable to say "the unique scalars $c_1, \dots, c_n$", as they are in fact unique!

Now I'd like to mention a few ways I think of $[x]_\beta$

- as instructions for recreating x if all you have access to is β
- as a "translation" of x into the "language" of β
  - this is incredibly informal, but helps me keep track of change of coordinate matrices
- if $\beta = \{v_1, v_2\}$ is a basis for $\mathbb{R}^2$ and I use v_1 and v_2 to create a skewed looking grid system on the plane (like the way I can use e_1 and e_2 to draw grid lines normally), then $[x]_\beta$ is the readout *using the weird new grid system* of where x is

- as an example, look at $\beta = \{(1,1),(0,1)\}$. Mark the y-axis at

  $$\dots,(0,-1),(0,0),(0,1),(0,2),(0,3),\dots$$

  and from each of those dots, draw lines in the $(1,1)$ direction. Then mark off points for each $(1,1)$ you travel along these lines and complete the whole grid.

  Then if I have a vector like $(3,5) \in \mathbb{R}^2$, I can see where these grid lines intersect it and notice that it's 3 steps in the $(1,1)$ direction and 2 steps in the $(0,1)$ direction. In other words, $[(3,5)]_\beta = (3,2)$.

  All this would be easier with a picture, but I'm not very fluent with TikZ (a common package for drawing things in LaTeX).

1.3 Examples of Coordinates

Let's do an example in $P_2(\mathbb{R}) = \{a + bx + cx^2 | a, b, c \in \mathbb{R}\}$.

We can check that $\beta = \{1 + x, x - x^2, x^2\}$ is a basis for P_2 (but I won't check here). Let's take another polynomial, $p = 1+2x+4x^2$ and express it in this basis!

$$\begin{aligned}[p]_\beta &= (c_1, c_2, c_3)\\ p &= c_1(1+x) + c_2(x - x^2) + c_3(x^2)\end{aligned}$$

this gives us equations to solve, since the coefficients of x^i must be equal for each i for these to be the same polynomial.

We get $c_1 = 1$, so $c_2 = 1$, which leaves us with $c_3 = 5$.

Thus $[p]_\beta = (1,1,5) \in \mathbb{R}^3$. We essentially "converted" a vector in P_2 into a vector in $\mathbb{R}^3$. This "conversion" smells like an isomorphism to me.

1.4 The coordinate function is an isomorphism

Let V be an n-dimensional vector space over F and β a basis for V. Define $f_\beta : V \to F^n$ by $f_\beta(x) = [x]_\beta$.

Then f_β is an *isomorphism* from V to F^n. Let's check:

- is f_β linear?
  - Yeah! It takes a bit of work to show, but it's definitely true!
- is f_β invertible?
  - Yes! If we know $[x]_\beta$, can we find x again? Yes! x is just found using $[x]_\beta$ as coefficients for a linear combination of the basis vectors!

- is f_β^{-1} also linear?
  - Just kidding, we don't need to check this. If we know the first two properties, this one always follows for free, so it's definitely true!

And keep in mind, we can pick a different basis to get a different isomorphism. So we have *lots* of isomorphisms from V to F^n now!

This is really convenient because some vector spaces look weird, and F^n is really simple. Why deal with $P_2(\mathbb{R})$ when we can deal with $\mathbb{R}^3$ instead?

2 Matrices vs Linear Transformations

2.1 Converting from Matrices to Linear Transformations

A matrix is just a rectangle of numbers, but we often want to multiply matrices by vectors.

Definition 2.1. Given an $m \times n$ matrix A with elements in F, define the *left-multiplication transformation* to be a linear map $L_A : F^n \to F^m$ given by

$$L_A(x) = Ax$$

This defines a *function* using a matrix, but $L_A \neq A$. They are different types of objects. One's a rectangle full of numbers, the other is a function. Also note that L_A is in fact a *linear function*! This is great news for us, because linear functions are fun.

However, sometimes people don't distinguish very carefully between A and L_A, so you'll need to be careful that you recognize the difference.

L_A **of** a vector is A **times** that vector. It makes sense to talk about the domain and codomain of the function L_A, but *not* of A.

Often times if we have a definition that applies to linear transformations, then we say that definition applies to matrices too. The *kernel* of a linear transformation is the set of vectors which map to zero, i.e., for a linear transformation $T : V \to W$, we have $\ker(T) = \{x \in V \mid T(x) = 0\}$. Sometimes people define the *kernel* of a matrix using the fact that we can get linear transformations from matrices: they'll say the kernel of an $m \times n$ matrix A is the kernel of L_A! Here that means $\ker(A) = \{x \in F^n \mid L_A(x) = 0\} = \{x \in F^n \mid Ax = 0\}$.

The point here is to **be careful**! Always keep clear whether you're talking about a matrix or a linear transformation, and be weary when definitions apply to only linear transformations, only matrices, or both!

What if we start with a linear transformation. How can we get a matrix from it?

2.2 Matrix of a linear transformation

2.2.1 Standard Matrix

First, suppose $T : F^n \to F^m$ is a linear transformation. The vector space F^n is pretty nice, because it has a really nice *standard basis* $\{e_1, \ldots, e_n\}$ where $e_1 = (1, 0, 0, \ldots, 0)$, $e_2 = (0, 1, 0, \ldots, 0)$, and so on.

A natural question is this: is $T = L_A$ for some A? We certainly need the domain and codomain for T to be F^n and F^m, because *any* L_A goes from F^n (for some n) to F^m (for some m). It doesn't act on other weirder vector spaces like P_3 or other things.

So assuming $T : F^n \to F^m$ is linear, is it L_A for some A? **Yes!** And even better, we can find it!

Proposition 2.2. *If $T : F^n \to F^m$ is linear, then $T = L_A$ for some matrix A called the "standard matrix of T". A is given by the formula:*

$$A = [T(e_1)|T(e_2)|\ldots|T(e_n)]$$

where those bars represent that the ith column of A is given by the vector $T(e_i) \in \mathbb{R}^m$.

As an example, if $T : \mathbb{R}^2 \to \mathbb{R}$ is given by $T(x) = x \cdot (1, 2)$, then we can find that $T = L_A$ where

$$\begin{aligned} A &= [T(e_1)|T(e_2)] \\ &= \begin{bmatrix} 1 & 2 \end{bmatrix} \end{aligned}$$

since $T(e_1) = 1$ and $T(e_2) = 2$. Thus we expect $T(x) = \begin{bmatrix} 1 & 2 \end{bmatrix} x = L_A(x)$ and we can check that this is indeed true!

To wrap this all up, we see that if A is the standard matrix of T, then L_A gives us T back again! We successfully have a one to one correspondance between linear maps from F^n to F^m and $m \times n$ matrices!

2.2.2 More General Formula

What about if $T : V \to W$ where V and W are just arbitrary vector spaces that are *not* F^n for some n?

Then it's *impossible* for $T = L_A$ for some A! Absolutely impossible! If A is $j \times k$ sized, then the map L_A has domain F^k and codomain F^j.

So what else can we do? We still want to represent linear transformations *somehow* with matrices, because rectangles full of numbers are fantastic (and we can do Gaussian elimination and things like that to get actual computable answers to things).

We can use **coordinates.** The vector $x \in V$ isn't in F^n (because V and F^n are different spaces), but what about $[x]_\beta$ when β is some basis for V? That's in F^n (where $\dim V = n$)! So we can define a matrix that multiplies by the coordinate vectors! We define it:

Proposition 2.3. *Let $T : V \to W$ be a linear map where $\dim V = n$ and $\dim W = m$. Suppose β is a basis for V and γ is a basis for W. Then T "can be represented by a matrix" A by which we mean there is some $m \times n$ matrix A such that*

$$A[x]_\beta = [T(x)]_\gamma$$

The matrix A is denoted $[T]_\beta^\gamma$ and can be calculated with the following column-by-column formula where $\beta = \{v_1, \ldots, v_n\}$:

$$[T]_\beta^\gamma = [[T(v_1)]_\gamma | \ldots | [T(v_n)]_\gamma]$$

where again the bars represent that the ith column of $[T]_\beta^\gamma$ is $[T(v_i)]_\gamma$.

Remember, the old basis vectors go into T, then you take coordinates with respect to the new basis.

Essentially we have two parallel pictures:

1. $T : V \to W$
2. $L_{[T]_\beta^\gamma} : F^n \to F^m$

These are "parallel" in the sense that anything in the second line is just the "coordinate" version of the first line. This phrasing is a bit vague but I hope it helps. We could write $T(x) = w$ or write $L_{[T]_\beta^\gamma}([x]_\beta) = [w]_\gamma$ and the same information is conveyed. Note that $L_{[T]_\beta^\gamma}([x]_\beta) = [T]_\beta^\gamma [x]_\beta$.

2.2.3 Back to the standard case

Let's go back to F^n for a minute and deal with a special case in a new way.

Suppose $x \in F^n$ and $\beta = \{e_1, \ldots, e_n\}$ is the standard basis. What's $[x]_\beta$?

Well if $x = (x_1, \ldots, x_n)$, then $x = x_1 e_1 + \cdots + x_n e_n$, so the coordinate vector is $(x_1, \ldots, x_n)$. But that's just x!

So if β is the *standard basis for F^n*, then $[x]_\beta = x$.

With this in hand, we can talk about the matrix of a linear $T : F^n \to F^m$ with respect to the standard bases. Here suppose β is the standard basis for F^n and γ is the standard basis for F^m. What does this give us?

Well, the definition says

$$[T]_\beta^\gamma [x]_\beta = [T(x)]_\gamma$$

but these are all *the standard basis* (on their respective spaces), so the above formula **in this special case** just says

$$[T]_\beta^\gamma x = T(x)$$

or in other words, if I call $A = [T]_\beta^\gamma$, I see $T(x) = Ax$! This means $T = L_A$! Cool!

This means that the more general way we described of finding a matrix given a linear transformation **lines exactly up with the more specific way we first talked about** when the linear transformation goes from F^n to F^m and both the bases are standard bases.

Note that these **won't** be the same if the bases chosen are **not** both the standard basis. And if T has a domain or a codomain which isn't F^k for some k, then there is **no way for** $T = L_A$ **to be possible**.

2.3 Change of basis

What if you have $[x]_\beta$ but you want $[x]_\gamma$? You can always take $[x]_\beta$ (and β) to find x, then try to find the coordinates of $[x]_\gamma$, but this takes a while.

I don't want to come up with a new formula though, so let's use the tools we already have!

We want a matrix that, when you multiply it by $[x]_\beta$, you get $[x]_\gamma$. Is there a linear transformation in here somewhere? Yes!

In fact, this matrix we want is exactly $[\mathrm{id}]_\beta^\gamma$, where id is the identity function $\mathrm{id}(x) = x$. Note that sometimes id_V is written I_V (the subscript tells us which space is the domain and codomain, since we can have different identity functions on different sets). Let's check this matrix works!

Using the formula that the matrix of a linear transformation satisfies, we get:

$$[\mathrm{id}]_\beta^\gamma [x]_\beta = [\mathrm{id}(x)]_\gamma$$

but $\mathrm{id}(x) = x$, so we just get $[x]_\gamma$! That's what we wanted!

Definition 2.4. The *change of basis matrix* on V from β to γ is the matrix $[\mathrm{id}]_\beta^\gamma$.

It's called the change of basis matrix because $[\mathrm{id}]_\beta^\gamma [x]_\beta = [x]_\gamma$ and so multiplying by it changes vectors in β coordinates to those same vectors in γ coordinates.

If $\beta = \{v_1, \dots, v_n\}$ we can use the column-by-column formula for the matrix of a linear transformation to write

$$\begin{aligned}[\mathrm{id}_V]_\beta^\gamma &= [[\mathrm{id}_V(v_1)]_\gamma | \dots | [\mathrm{id}_V(v_n)]_\gamma] \\ &= [[v_1]_\gamma | \dots | [v_n]_\gamma]\end{aligned}$$

Also fun fact: if A is the change of basis matrix from β to γ, then A^{-1} is the change of basis matrix from γ to β.

This is a special case of the fact that *composition of linear transformations and multiplication of matrices are related.*

2.4 Composition and multiplication

2.4.1 Easier Case

First let's deal with the easier case. If $T : F^n \to F^m$ and $S : F^m \to F^p$ are linear, then we can define the linear function $S \circ T$. This is the *composition*, and it's defined by $(S \circ T)(x) = S(T(x))$.

But since $T : F^n \to F^m$ is linear, it has a standard matrix A $(m \times n)$ such that $T = L_A$. And S has a $p \times m$ sized matrix B such that $S = L_B$.

- What's the standard matrix of $S \circ T$?
- What is L_{BA}?

The answer to both of these questions is the fact that **matrix multiplication corresponds to composition of linear transformations**:

$$S \circ T = L_{BA}$$

or in other words,

$$L_{BA} = L_B \circ L_A$$

multiplication of the matrices B and A leads to *composition* of their corresponding linear transformations!

2.4.2 General Case

As before, the above expression is really a special case of what we can do with coordinates.

Suppose $T : V \to W$ and $S : W \to U$. Let α, β, γ be bases for U, V, W respectively. Then

$$[S]^{\alpha}_{\beta}[T]^{\beta}_{\gamma} = [S \circ T]^{\alpha}_{\gamma},$$

or in other words, **composition of linear transformations corresponds to the product of their matrices with respect to given bases**.

Know what S and T do to coordinates, but need to understand $S \circ T$? Well, just multiply the matrices (as long as the basis for the middle space is the same, so like $[S]^{\alpha}_{\beta}[T]^{\delta}_{\gamma}$ will just be nonsense).

So what does this have to do with the L_A stuff from the previous section? Well, remember that if $T = L_A$, then A is just the matrix of T **with respect to the standard bases.** So we can turn everything in the previous section about BA into $[S]^{\text{standard}}_{\text{standard}}[T]^{\text{standard}}_{\text{standard}}$ for standard bases on the right spaces.

3 Eigenvalues and Eigenvectors

3.1 What are eigenvalues and eigenvectors?

Let $T : V \to V$ be linear. The function T might do all sorts of weird things to vectors, sending some of them to 0, stretching others, rotating others.

If $T(x)$ is just a *scaled* copy of x, and x is non-zero, we say that x is an eigenvector! Let's be more precise:

Definition 3.1. Let $T : V \to V$ be linear and $x \neq 0$. If $T(x) = \lambda x$ for some scalar λ, then we say x is an *eigenvector* with *eigenvalue* λ.

The eigenvalue is the amount x will stretch by under the function T. Note that this can be negative! So if $T(x) = -x$ (and $x \neq 0$), then x is still an eigenvector with eigenvalue -1.

3.2 Characteristic polynomial and how to find eigenvalues

3.2.1 The setup

If $T(x) = \lambda x$, then $T(x) = \lambda\,\mathrm{id}(x)$, or in other words $(T - \lambda\,\mathrm{id})(x) = 0$, so $x \in \ker(T - \lambda\,\mathrm{id})$ where id is the identity function on V. And this works in reverse, too!

This means

Proposition 3.2. *If $T : V \to V$ is linear and $x \neq 0$, then x is an eigenvector with eigenvalue λ if and only if $x \in \ker(T - \lambda\,\mathrm{id})$.*

So when can we find such an x? Whenever $\ker(T - \lambda \,\mathrm{id})$ has non-zero vectors in it! And remember that $(T - \lambda \,\mathrm{id}) : V \to V$, and a linear transformation from V to *itself* is an isomorphism iff it's a surjection iff it's an injection (whenever $\dim V$ is finite!).

(**Warning:** $T : V \to W$ is an isomorphism iff it's an injection iff it's a surjection is **false** if $\dim V \neq \dim W$ or if either dimension is infinite!)

Remember that $T - \lambda \,\mathrm{id}$ is *injective* if and only if the kernel is $\{0\}$.

This means we just need to find when $T - \lambda \,\mathrm{id}$ is *non-invertible* to discover when T has some eigenvector with eigenvalue λ. And determinants are really good for checking when something is invertible!

Let's convert our discussion now to matrices: if A is an $n \times n$ matrix, then we'll say A has an eigenvector x with eigenvalue λ if L_A does. That means we're looking at $Ax = \lambda x$ now. The argument above works just the same. Now with matrices we can do calculations!

Proposition 3.3. *A scalar λ is an eigenvalue of the $(n \times n)$ matrix A (that is to say, there is a non-zero x such that $Ax = \lambda x$), if and only if* $\det(A - \lambda I_n) = 0$*, where I_n is the $n \times n$ identity matrix.*

This tells us how to find eigenvalues!

3.2.2 How to find eigenvalues:

- If A is a matrix, calculate $\det(A - \lambda I_n)$. This will be a polynomial called the **characteristic polynomial of** A

- Set the characteristic polynomial equal to 0 and find the solutions. These are the roots of the characteristic polynomial, and they are our eigenvalues!

Definition 3.4. If a polynomial factors complete, we say it **splits**. Not every polynomial splits when the field is $\mathbb{R}$, for example $x^2 + 1$ can't be factored! But when the field is $\mathbb{C}$, *every* polynomial can be factored (this is the "fundamental theorem of algebra").

3.3 Finding eigenvectors

How can we find eigenvectors? Well, first find the eigenvalues. Then if λ is an eigenvalue of a matrix A, just calculate $\ker(A - \lambda I)$. This will contain 0, and *everything else in this space is an eigenvalue!*

This is a subspace, so it's going to have infinitely many vectors in it!

Definition 3.5. The *eigenspace* to λ for a matrix A is the space $E_\lambda = \ker(A - \lambda I)$

Often we might want to find a basis for E_λ when λ is an eigenvalue.

3.4 Multiplicities

When we factor the characteristic polynomial, we'll get something like $(\lambda - 1)^2(\lambda - 3)^4(\lambda - 0)^3$.

We often want to talk about those exponents. Those are the *algebraic multiplicities*. In this example, the algebraic multiplicity of 1 is 2, the algebraic multiplicity of 3 is 4, and the algebraic multiplicity of 0 is 3.

When we move on to calculating eigenvectors, we get these eigenspaces E_λ. Often we want to talk about their dimension. The dimension $\dim E_\lambda$ is called the *geometric multiplicity* of λ.

Proposition 3.6. *The algebraic multiplicity of λ is always* greater than or equal to *the geometric multiplicity.*

This will be useful when we talk about diagonalization!

3.5 Example

Let's define a 3×3 matrix with $\mathbb{R}$ valued entries as follows:

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}.$$

Let's calculate everything!

- We want to find eigenvalues first.

  First we calculate $\det(A - \lambda I)$, the characteristic polynomial. This is

$$\begin{aligned} \det\left(\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right) &= \det\left(\begin{bmatrix} 1-\lambda & 0 & 1 \\ 0 & 2-\lambda & 0 \\ 1 & 0 & 1-\lambda \end{bmatrix}\right) \\ &= -\lambda^3 + 4\lambda^2 - 4\lambda \\ &= (\lambda - 0)(\lambda - 2)^2 \end{aligned}$$

  This means we have two eigenvalues: 0 is an eigenvalue with algebraic multiplicity 1 and 2 is an eigenvalue with algebraic multiplicity 2.

- Now eigenvectors:

  Skipping some computation, we calculate $E_0 = \ker(A - 0I) = \operatorname{span}\{(-1, 0, 1)\}$. This is a dimension 1 subspace of $\mathbb{R}^3$, so $\dim E_0 = 1$ and the geometric multiplicity of 0 is 1.

  We also calculate $E_2 = \ker(A - 2I) = \operatorname{span}\{(1, 0, 1), (0, 1, 0)\}$, which is a dimension 2 subspace of $\mathbb{R}^3$, so $\dim E_2 = 2$ and the geometric multiplicity of 2 is 2.

4 Diagonalization

I don't have enough time to write this up so let me just include some questions and a few statements.

4.1 Diagonalize a linear transformation

- If a linear transformation T is diagonalizable, that means there is some basis β such that $[T]_\beta^\beta$ is a diagonal matrix.
  - Will it be diagonal for *every* basis?
  - What's special about a basis that makes $[T]_\beta^\beta$ diagonal, and what does this have to do with eigenvalues or eigenvectors?
  - If $[T]_\beta^\beta$ is a diagonal matrix, *what* diagonal matrix is it? What does this have to do with eigenvectors or eigenvalues?
- *When* is a linear transformation diagonalizable? Always? Sometimes?
  - If the characteristic polynomial doesn't split, what does this mean?
  - If a single eigenvalue has geometric multiplicity *strictly less* than its algebraic multiplicity, what happens?

4.2 What does this mean in the matrix world?

- We say a matrix A is diagonalizable if and only if L_A is diagonalizable as a linear transformation. Can I express this only with matrices though? What does $A = SDS^{-1}$ represent?
- If I write $A = SDS^{-1}$, how can I interpret S as a change-of-coordinates matrix?

- What will D contain on its diagonal?
- What will S contain as its columns?

4.3 Calculations!

- We already worked out the eigenvectors and eigenvalues for a matrix in a previous section. Can you diagonalize that matrix?